**Chapter 12 –Basic responses of Second-Order Circuits**

**P12.1.1** Determine *β* in Figure P12.1.1 so that the response is critically damped by applying: (i) the source absorption theorem, and (ii) KVL and comparing coefficients with those of a prototypical series *RLC* circuit.



**Solution:**  rad/s; For critical damping, ,  Ω.

1. The effective resistance of the source is *βvR*/(*vR*/100) = 100*β.* Hence, 100*β* + 100 = 160, which gives *β* = 0.6.
2. From KVL, with *vSRC* = 0:  + , where *vR* = 100*i*. The effective resistance is therefore (*β* + 1)×100 Ω. For critical damping, ,  Ω; (*β* + 1)×100 = 160; hence, *β* = 0.6.



**P12.1.8** The switch in Figure P12.1.8 is opened at *t* = 0 after being closed for a long time. (a) Determine *iL*, *vC*, *dvC*/*dt*, and *diL*/*dt* just after the switch is opened. (b) Is the circuit reducible to a prototypical series *LCR* circuit or a parallel *GCL* circuit?

**Solution:** Just before *t* = 0, *iC* = 0 and the3 mA divides between the 3 kΩ and 6 kΩ resistors, so that *IL*0 = 3×3/9 = 1 mA; *VC*0 = 6×1 = 6 V. These do not change on switching. Just after *t* = 0, 2*iC* + *iL* = 0, so that *iC* = -*iL*/2 = -0.5 mA = *C*, and **-1000 V/s. From KVL, *VC*0 + 1×*iC* = 6*IL*0 + *vL*. Hence, *vL* = 6 – 0.5 – 6 = -0.5 V = *L*, and **=** = -5 A/s.

**P12.2.1** Given  mA in Figure P12.2.1. Determine , , *vL*, and *vC*, assuming zero initial charge on the capacitor. Verify that *vR* + *vL* + *vC* = 0.



**Solution:** It is seen from the expression for current that the response is critically damped;  rad/s; hence H; *R* = 2×6.25×400 = 5 KΩ; =  + =  V; =  V. As a check, .



**P12.2.4** Given *v* = 0, *I*10 = *I*20 = 2 A at *t* = 0. Determine the total instantaneous energy in the inductors as a function of time

**Solution:** Since *v* = 0 at *t* = 0, it means that at *t* = 0, *i* is a maximum and is given by *i* = 4cos*ωt* A, where ** rad/s. The energy in the inductors is therefore ** J.

**P12.2.7** The switch in Figure P12.2.7 is moved



from position ‘a’ to position ‘b’ at *t* = 0 after being in position ‘a’ for a long time. Determine *vC* and *iL* for *t* ≥ 0.

**Solution:**  rad/s,  rad/s; response is overdamped.  rad/s; . At *t* = 0, *iL* = 2 A, *vC* = 0. . At *t* = 0, *A* + *B* = 2.  . At *t* = 0, . Solving for A and B gives: , and . It follows that  A, where *t* is in ms and  kV.

**P12.2.12** The switch in Figure P12.2.12, is moved to position ‘b’ at *t* = 0, with zero initial energy storage. Determine: (a) for critical damping and (b) , , and  for



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**Solution:** (a) For critical damping *R* = , where *ω*0 =   10 krad/s; *R* =  = 50 Ω. To have the parallel resistance equal to 50Ω, *ρ* = 0.

(b) Just before the switch is moved, *v* = 0 and *iL* = 0. Just after the switch is moved, these remain the same and *iC* = 0.1 A. For *t* ≥ 0, ; hence, *A* = 0. . It follows that V/s, or 100 V/ms. Hence, *v* = 100*te*-10*t* V, where *t* is in ms; *i*C = A, *t* being in seconds, or *iC* = 100(*e*-10*t* – 10*te*-10*t*) mA, *t* being in ms. Since *iL* + *iC* + mA, *iL* = 100 – 100(*e*-10*t* – 10*te*-10*t*) mA, *t* being in ms, or *iL* = 100 –100(*e*-10*t* + 10*te*-10*t*) mA, *t* being in ms. As a check, A, with *t* in ms, or = mA. Integration gives the same answer.

**P12.2.14** The switch In Figure P12.2.14, is moved to position ‘b’ at  after being in position ‘a’ for a long time. Determine  for *t* ≥ 0.



**Solution:** The initial current through the inductor is  10mA, and the initial voltage across the capacitor is 5 V. After the switch is moved to position b, the circuit becomes a parallel circuit having *C* = 1μF, *L* = 4 H, and *Rp* = 500 Ω. Hence,  krad/s, rad/s; V, where  krad/s,  krad/s and *t* is in ms.   mA, where if *vO* is in V, *t* is ms and *L* is in henries, *i* is in mA. At *t* = 0, *A + B* = -5 and  mA. This gives:

 V and  V. It follows that  V.

**P12.2.18** The switch in Figure P12.2.18 is opened at *t* = 0 after being closed for a long time. Determine *vL* for *t* ≥ 0.



**Solution:**  rad/s.  rad/s. response is underdamped, with  rad/s. Just before the switch is opened, *vL* = 0 and *iL* = 2 A. Just after the switch is opened, *vL* = 0, *iL* = 2 A, and *iC* = -2 A.



. At t = 0, *vL* = 0, so *A* = 0, and . , retaining only the cosine term. At *t* = 0, *iC* = -2 A, so that *B* = *iC*/*ωdC* =  V. it follows that  V, where *t* is in ms.